Dissipation-driven phase transitions in superconducting wires

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We report on the reinforcement of superconductivity in a system consisting of a narrow superconducting wire weakly coupled to a diffusive metallic film. We analyze the effective phase-only action of the system by a perturbative renormalization group and a self-consistent variational approach to obtain the critical points and phases at $T=0$. We predict a quantum phase transition toward a superconducting phase with long-range order as a function of the wire stiffness and coupling to the metal. We discuss implications for the dc resistivity of the wire.

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I. INTRODUCTION

The interplay between fluctuation and dissipation phenomena in quantum systems is presently under intensive research. Fluctuations are particularly strong in low dimensions, as reflected by the lack of long-range order in one-dimensional (1D) systems with short-range interactions.[1,2] On the other hand, dissipation counteracts fluctuations effects, decreasing the lower critical dimension.[3–6]

Some physical realizations of dissipative low-dimensional systems are the well-known resistively shunted Josephson junctions arrays, where the effect of local ohmic dissipation has been intensively studied,[7–11] superconducting grains embedded in metallic films,[12–15] and Luttinger liquids coupled to dissipative baths.[16–18]

Narrow superconducting wires with diameter $d \ll \xi_0$ (where ξ_0 is the bulk superconducting coherence length) are low-dimensional systems in which strong fluctuations of the order parameter affect low-temperature properties.

It was originally suggested by Little,[19] and subsequently discussed by Langer and Ambegaokar (LA) (Ref. 20) and McCumber and Halperin (MH) (Ref. 21) that resistivity $\varrho(T)$ in thin wires would be finite for all temperatures below the bulk critical temperature T_c. Thermal fluctuations cause the magnitude of the order parameter to temporarily vanish at some point along the wire, allowing its phase to slip by 2π (the so-called thermally activated phase slips) and dissipate through the Josephson relation $V=\hbar/2e\ \Delta\dot{\theta}$, where $\Delta\theta$ is the phase difference across the wire.

According to the LA-MH theory, thermal fluctuations induce a resistivity $\varrho(T) \sim \Omega(T)\exp[-\Delta F_0/T]$, where ΔF_0 is the Ginzburg-Landau free-energy barrier between different current-carrying states in the wire and $\Omega(T)$ is an algebraically decreasing function of T. However, deviations from the LA-MH theory in the regime $T \ll T_c$ were first observed by Giordano[22,23] and more recently by other experimental groups,[24–28] leading to the conclusion that for very thin wires at low temperatures current decay was produced by macroscopic quantum tunneling of the phase of the order parameter through the same free-energy barriers (the so-called quantum phase slips), leading to a much weaker dependence of the resistivity on T.

Moreover, it is believed that the destruction of the superconducting state in very thin wires occurs through the proliferation of quantum phase slips/antiphase slips pairs,[23,24,27,29–33] in what constitutes the quantum analog in $1+1$ dimensions to the classical Berezinskii-Kosterlitz-Thouless (BKT) (Ref. 34) transition in two dimensions (2D).

Contrary to other 1D systems such as dissipative ohmic Josephson-junction arrays, isolated thin wires do not present significant sources of dissipation at $T \ll T_c$.[30,33] However, additional sources might be provided by a coupling to the environment, a possibility which has hardly been explored yet. Although general theoretical frameworks have been proposed to describe superconductor-normal (SN) junctions,[13,35,36] recent advances in superconducting nanowires fabrication techniques call for a more detailed analysis of the effects of coupling to general dissipation sources.[22–24,27,31]

In this paper we focus on the effect of weakly coupling a superconducting wire to a diffusive 2D normal metal. We show how the induced dissipation stabilizes superconductive long-range order at $T=0$ despite the 1D nature of the wire. At finite T, the effect of dissipation are manifested in an increase in the superconductive stiffness of the wire.

The paper is organized as follows: in Sec. II we derive the effective low-energy phase-only action of the coupled system. Section III is devoted to the analysis of this model

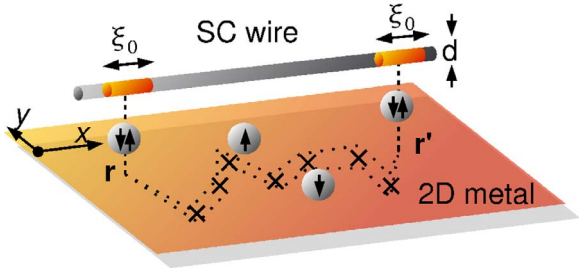

FIG. 1. (Color online) Representation of the system. At $T \ll T_c$ one-particle hopping is suppressed by the BCS gap-energy Δ_0. At next order in the hopping process, Cooper pairs can tunnel into the metal and propagate coherently in a length ξ_N, generating an effective coupling $\sim\cos[\theta(\boldsymbol{r})-\theta(\boldsymbol{r}')]$ in the wire.

within a perturbative renormalization group (RG) and a self-consistent harmonic approximation and discuss implications for the dc resistivity. Finally, in Sec. IV we discuss the main physical consequences of our results and summarize them.

II. MODEL

We analyze the system depicted in Fig. 1, which represents a clean superconducting wire of length L and lateral dimensions $d \ll \xi_0$, weakly coupled to a diffusive 2D metal. In the following we use the convention $\hbar = k_B = 1$. We begin our description with the action of the microscopic BCS Hamiltonian for the isolated wire

$$S_{\mathrm{w}} = \int_0^\beta d\tau \int d^3R \sum_\sigma \left\{ \bar{\psi}_\sigma (\partial_\tau - \mu)\psi_\sigma + \frac{[\nabla \bar{\psi}_\sigma][\nabla \psi_\sigma]}{2m} \right\} + U \int_0^\beta d\tau \int d^3R \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow . \quad (1)$$

Here the fermionic field $\psi_\sigma \equiv \psi_\sigma(\boldsymbol{R},\tau)$ describes an electron in the wire with spin projection σ at position $\boldsymbol{R} \equiv (x,y,z)$ and imaginary time τ. The chemical potential $\mu = k_F^2/2m$ is the Fermi energy in the normal state and the local attractive interaction $U<0$ is responsible for pairing at $T<T_c$.

Assuming for simplicity that the coupling to the metallic film takes place along the line $(x,0,0)$ in the wire, the coupling term is described by

$$S_\perp = t_\perp \int_0^\beta d\tau \int dx \sum_\sigma [\bar{\psi}_\sigma(x,\tau)\eta_\sigma(x,\tau) + \mathrm{H.c.}], \quad (2)$$

where the fermionic field $\eta_\sigma(\boldsymbol{r},\tau)$ represents an electron at position $\boldsymbol{r} \equiv (x,y)$ in the film. Here the compact notations $\psi_\sigma(x,\tau) \equiv \psi_\sigma(\boldsymbol{R},\tau)|_{y=z=0}$ and $\eta_\sigma(x,\tau) \equiv \eta_\sigma(\boldsymbol{r},\tau)|_{y=0}$ have been used. While certainly more realistic models for the coupling, which take into account geometrical details of the SN junction have been studied,[13,35,36] the main physics which is of interest to us is already captured by Eq. (2).

Electronic motion in the metallic film is described by the noninteracting action

$$S_{2\mathrm{D}} = \int_0^\beta d\tau \int d^2r \sum_\sigma \left\{ \bar{\eta}_\sigma (\partial_\tau - \mu_{2\mathrm{D}})\eta_\sigma + \frac{[\nabla \bar{\eta}_\sigma][\nabla \eta_\sigma]}{2m} + V(\boldsymbol{r})\bar{\eta}_\sigma(\boldsymbol{r})\eta_\sigma(\boldsymbol{r}) \right\}, \quad (3)$$

where $V(\boldsymbol{r})$ is the (static) disorder potential. We assume weak enough disorder, such that the localization length in the film is $\xi_{\mathrm{loc}} \gg L$, allowing us to neglect strong localization effects.

For one given realization of the disorder potential $V(\boldsymbol{r})$, the effective action in the wire is obtained by integrating the electronic degrees of freedom in the metallic film

$$S_{\mathrm{w}}^{\mathrm{eff}} = S_{\mathrm{w}} + S_{\mathrm{diss}},$$

$$S_{\mathrm{diss}} = -t_\perp^2 \int_0^\beta d\tau d\tau' \int dx dx' \times \sum_\sigma \bar{\psi}_\sigma(x,\tau) g_{2\mathrm{D}}(x,x';\tau-\tau')\psi_\sigma(x',\tau'),$$

where $g_{2\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}';\tau-\tau')$ is the Green's function in the film. Note that the spin index has been dropped using the SU(2) symmetry of the problem and that we used the notation $g_{2\mathrm{D}}(x,x';\tau-\tau') \equiv g_{2\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}';\tau-\tau')|_{y=y'=0}$.

Since the disorder potential breaks the original translation invariance in the wire, an average over different configurations of the disorder is needed to restore it. Let us define the partition function of the systems for one disorder realization as

$$Z[V] \equiv \int \mathcal{D}[\psi] e^{-S_{\mathrm{w}} - S_{\mathrm{diss}}},$$

Assuming for convenience that $V(\boldsymbol{r})$ is Gaussian distributed

$$S_{\mathrm{d}} = \frac{1}{2\mathcal{V}^2} \int d^2r V^2(\boldsymbol{r}),$$

we can formally perform the average over different disorder configurations as

$$Z = \frac{\int \mathcal{D}[V] e^{-S_{\mathrm{d}}} Z[V]}{\int \mathcal{D}[V] e^{-S_{\mathrm{d}}}}.$$

Expansion of $Z[V]$ in powers of $t_\perp$ allows us to obtain an explicit form of the partition function Z

$$Z = \frac{\int \mathcal{D}[V] e^{-S_{\mathrm{d}}} \int \mathcal{D}[\psi] e^{-S_{\mathrm{w}}} \sum_{n=0}^{\infty} \frac{1}{n!} S_{\mathrm{diss}}^n}{\int \mathcal{D}[V] e^{-S_{\mathrm{d}}}},$$

$$= \int \mathcal{D}[\psi] e^{-S_{\mathrm{w}}} \sum_{n=0}^{\infty} \frac{1}{n!} \langle S_{\mathrm{diss}}^n \rangle_{\mathrm{d}}. \quad (4)$$

The low-energy effective action of this model is obtained by introducing Hubbard-Stratonovich fields $\Delta(\boldsymbol{R},\tau), \Delta^*(\boldsymbol{R},\tau)$ in the particle-particle channel to decouple the quartic term in S_{w}.[30,33,37,38] After integration of the fermionic degrees of freedom in the wire, the action reads

$$S_{\mathrm{w}}[\bar{\Delta},\Delta] = -\mathrm{Tr}\ \ln \boldsymbol{g}_{\mathrm{w}}^{-1} - \frac{1}{U}\int_0^\beta d\tau \int d^3R |\Delta(\boldsymbol{R},\tau)|^2, \quad (5)$$

where

$$\boldsymbol{g}_{\mathrm{w}}^{-1} \equiv \begin{bmatrix} \partial_\tau - \mu - \dfrac{\nabla^2}{2m} & \Delta(\boldsymbol{R},\tau) \\ \bar{\Delta}(\boldsymbol{R},\tau) & \partial_\tau + \mu + \dfrac{\nabla^2}{2m} \end{bmatrix}$$

and where the Nambu notation

$$\Psi(\boldsymbol{R},\tau)=\begin{pmatrix}\psi_{\uparrow}(\boldsymbol{R},\tau)\\ \bar{\psi}_{\downarrow}(\boldsymbol{R},\tau)\end{pmatrix}$$

is implicit. The Green's function in the wire formally reads

$$\boldsymbol{g}_{\mathrm{w}}(\boldsymbol{R},\tau)\equiv\begin{bmatrix}g(\boldsymbol{R},\tau) & f(\boldsymbol{R},\tau)\\ \bar{f}(\boldsymbol{R},\tau) & \bar{g}(\boldsymbol{R},\tau)\end{bmatrix},$$

where $g(\boldsymbol{R},\tau)\equiv\langle T_\tau\psi(\boldsymbol{R},\tau)\bar{\psi}(0)\rangle$ and $\bar{g}(\boldsymbol{R},\tau)\equiv\langle T_\tau\bar{\psi}(\boldsymbol{R},\tau)\psi(0)\rangle$ denote, respectively, the particle and hole propagators in the wire while $f(\boldsymbol{R},\tau)\equiv\langle T_\tau\psi(\boldsymbol{R},\tau)\psi(0)\rangle$ and $\bar{f}(\boldsymbol{R},\tau)\equiv\langle T_\tau\bar{\psi}(\boldsymbol{R},\tau)\bar{\psi}(0)\rangle$ are the anomalous ones.[39]

For very narrow wires with diameter $d\ll\xi_0$ at low energies, the dependence of the fields $\bar{\Delta}(\boldsymbol{R},\tau),\Delta(\boldsymbol{R},\tau)$ on transverse dimensions can be neglected, reducing to $\Delta(\boldsymbol{R},\tau)\to\Delta(\boldsymbol{x})$ where the compact notation $\boldsymbol{x}=(x,\tau)$ has been used. Moreover, at $T\ll T_{MF}$ [where T_{MF} is the mean-field critical temperature of Eq. (5)] and neglecting amplitude fluctuations, the dynamical state of the wire is characterized by $\Delta(\boldsymbol{x})=\Delta_0e^{i\theta(\boldsymbol{x})}$, where the quantity Δ_0 corresponds to the (temperature-dependent) BCS energy gap and $\theta(\boldsymbol{x})$ is the space-dependent and time-dependent phase of the macroscopic BCS wave function.

The derivation of the phase-only action in the isolated wire [i.e., the first term in the expansion of Eq. (4)] is obtained by the means of an expansion in Gaussian fluctuations in the gradients of the field $\theta(\boldsymbol{x})$ around the BCS saddle point and takes the form of a Luttinger liquid action[2,30,33,37,40]

$$S_0=\int d\boldsymbol{x}\left[-i\Pi\partial_\tau\theta+\frac{uK}{2\pi}(\nabla\theta)^2+\frac{u}{2\pi K}(\pi\Pi)^2\right]. \tag{6}$$

Here $\Pi\equiv\Pi(\boldsymbol{x})$ is the momentum canonically conjugate to $\theta(\boldsymbol{x})$, formally defined through the relation $[\theta(x),\Pi(x')]=i\delta(x-x')$ and representing fluctuations in the density of Cooper pairs at point $\boldsymbol{x}$. The operator ∇ denotes derivation with respect to the spatial coordinate x. The Luttinger liquid parameters u and K are defined as[2,30]

$$u\equiv\sqrt{\frac{A_{\mathrm{w}}n_s(T)}{4m\kappa(T)}},$$

$$K\equiv 2\pi\sqrt{\frac{A_{\mathrm{w}}n_s(T)\kappa(T)}{4m}},$$

where $\frac{n_s(T)}{4m}$ is the superconducting stiffness of the wire [with $n_s(T)$ the three-dimensional density of electrons in the condensate and m their mass], A_{w} is the cross-sectional area of the wire, and $\kappa(T)$ is the compressibility (cf. Ref. 30 for more details). u corresponds to the velocity of the plasma (Mooij-Schön[41]) mode. In the following, we assume the wire to be in the thermodynamic limit $L\gg L_T=u/T$.

The next terms in the expansion of the partition function [Eq. (4)] provide the effects of the coupling to the metallic film. At order $\mathcal{O}(t_\perp^2)$ and at low temperatures ($T\ll T_{MF}$), the transfer of individual electrons is strongly forbidden by the energy gap Δ_0, giving a probability $\sim e^{-\Delta_0/T}$ for such a charge-transfer channel.

The most relevant contribution of the coupling to the metallic film appears at order $\mathcal{O}(t_\perp^4)$ and corresponds to the Andreev reflection occurring at SN interfaces.[12,13,35,36,42] This contribution physically represents processes in which *paired* electrons (for which there is no energy cost) are effectively transferred from the wire to the film and vice versa[42]

$$S_A=\frac{1}{2}\langle S_{\mathrm{diss}}^2\rangle=2t_\perp^4\left[\int d\boldsymbol{x}''f(\boldsymbol{x}'')\right]^2\times\int d\boldsymbol{x}d\boldsymbol{x}'P_c(\boldsymbol{x}-\boldsymbol{x}')\cos[\theta(\boldsymbol{x})-\theta(\boldsymbol{x}')], \tag{7}$$

where $f(\boldsymbol{x})=f(\boldsymbol{R},\tau)\big|_{y=z=0}$. The kernel $P_c(\boldsymbol{x})=P_c(\boldsymbol{r},\tau)\big|_{y=0}$ is the cooperon propagator in the diffusive film, defined as[42,43]

$$P_c(\boldsymbol{r}-\boldsymbol{r}',\tau-\tau')\equiv\langle g_{2\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}',\tau-\tau')g_{2\mathrm{D}}(\boldsymbol{r},\boldsymbol{r}',\tau-\tau')\rangle_{\mathrm{d}}, \tag{8}$$

representing the probability to find a coherent *electron pair* traveling a distance $|\boldsymbol{r}-\boldsymbol{r}'|$ in the interval $\tau-\tau'$ through the disordered film[43] (see Fig. 1). The diffusive propagation of this electron pair remains phase coherent over a length ξ_N (assumed $\gg\xi_0$) which depends on T, magnetic field, and the strength of Coulomb interactions.[43] In the absence of the latter, $\xi_N(T)\simeq\sqrt{\frac{D}{T}}$, which leads to important nonlocal coupling effects at low enough temperatures.

Explicit evaluation of Eq. (8) for a diffusive 2D metal, assuming a Fermi-liquid description, yields (see Appendix)

$$P_c(\boldsymbol{r},\tau)\approx\frac{\rho_{2\mathrm{D}}}{2\pi^2D}\tilde{P}_c(\boldsymbol{r},\tau), \tag{9}$$

where we have defined

$$\tilde{P}_c(\boldsymbol{r},\tau)=\mathrm{Re}\left\{\frac{\exp\left(-\frac{r}{\xi_N}+\frac{ir^2}{4D\tilde{\tau}}\right)}{\tilde{\tau}^2}\Gamma\left(0,\frac{ir^2}{4D\tilde{\tau}}\right)\right\}.$$

Here $\Gamma(a,z)$ is the incomplete gamma function and $\tilde{\tau}\equiv\tau+i\tau_e$, with τ_e the elastic lifetime of electrons in the diffusive film.[43] Equation (9) is a valid expression for $\tau\gg\tau_e$ and $x\gg l_e$, where l_e is the elastic mean-free path in the film. In what follows, we set $y=0$ and consider the kernel as depending only on the coordinate $\boldsymbol{x}$.

The coherence length $\xi_N(T)$ separates two distance regimes of interest: (a) *the local regime* $x\gg\xi_N(T)$, where the cooperon can be considered local in space, reducing to

$$\tilde{P}_c(\boldsymbol{x})\approx\frac{\xi_N\delta(x)}{\tau^2}\ln\left(\frac{4D\tau}{\xi_N^2}\right)$$

consistent with the expression for the Andreev conductance in Refs. 35. Introducing the notation $\boldsymbol{q}\equiv(k,\omega_m)$, where ω_m is the bosonic Matsubara frequency $\omega_m=2\pi mT$, the approximated Fourier transform (neglecting the logarithm) is independent of k for $k<\xi_N^{-1}$ and reads

$$\tilde{P}_c(\boldsymbol{q})\approx 2\xi_N\left[\frac{1}{\tau_e}-\pi|\omega_m|\right]$$

in the limit $\boldsymbol{q}\to 0$.

(b) *The nonlocal regime* of distances $x < \xi_N(T)$, where Eq. (9) can be approximated as

$$\tilde{P}_c(\boldsymbol{x}) \approx \frac{(4D)^2}{x^4 + (4D\tau)^2} \tag{10}$$

with Fourier transform

$$\tilde{P}_c(\boldsymbol{q}) \approx 2\pi^2\sqrt{D}\left[\sqrt{\frac{\pi}{\tau_e}} - 2\sqrt{Dk^2 + |\omega_m|}\right]$$

for $\boldsymbol{q} \to 0$.

It is convenient to introduce the normal-state tunnel conductance per unit of length in the SN junction[44]

$$G_t = \left(\frac{h}{2e^2}\right)\left(\frac{1}{2\pi}\right)^2 t_\perp^2 \frac{\rho_w \Omega_w \rho_{2D} A_{2D}}{L},$$

where $\rho_w(\rho_{2D})$ is the normal-state local density of states in the wire (film) and Ω_w (A_{2D}) is the volume of the wire (area of the film). Replacing the expression of the cooperon appearing in Eq. (9) and noting that the resistivity in a 2D film is $\varrho_{2D} = [e^2 \frac{n_{2D}}{m} \tau_e]^{-1} = [e^2 \rho_{2D} D]^{-1}$, we can express Eq. (7) as

$$S_A = \frac{G_A}{\xi_0^2} \int d\boldsymbol{x} d\boldsymbol{x}' \tilde{P}_c(\boldsymbol{x} - \boldsymbol{x}') \cos[\theta(\boldsymbol{x}) - \theta(\boldsymbol{x}')], \tag{11}$$

where G_A is the dimensionless Andreev conductance in the SN junction[35,36]

$$G_A = \left(\frac{1}{2\pi}\right)^4 4e^2 G_t^2 \varrho_{2D}.$$

In addition to the term S_A of Eq. (7), the coupling $t_\perp$ generates contributions $\mathcal{O}(t_\perp^2)$ and $\mathcal{O}(t_\perp^4)$ at scales $x \lesssim \xi_0$ and $\tau \lesssim \xi_0 u^{-1}$, which renormalize the bare Luttinger parameters K and u of Eq. (6) (e.g., diffuson propagator[43]). Although these contributions do not change the physics at a qualitative level, their effect is relevant for the comparison with real systems. A microscopic study of the dependence of K and u on the hopping $t_\perp$, as well as further renormalization arising from Coulomb interactions between the wire and the film, is beyond the scope of the present paper and will be given elsewhere.[45] In the following we assume that the Luttinger parameters appearing in Eq. (6) already include all these corrections. Note also that the coupling to the metal modifies the bare value of Δ_0 through the well-known proximity effect in which the diffusion of normal electrons in the superconductor produce a lowering of T_c.[46] However, since this is a small effect of order $\mathcal{O}(t_\perp^4)$ and in addition we assume $T \ll T_c$, this effect is irrelevant to our description and can be effectively taken into account in renormalized values of Δ_0 and T_c.

So far we have not included the effects of topological defects (phase slips) in the wire. As discussed in Sec. I, these topological excitations produce finite resistivity at $T \ll T_c$ and are believed to be the origin of destruction of superconductivity in narrow wires[30,33] and in dissipative Josepshon junctions arrays.[10,11] It can be shown[2] that defining a field $\phi(\boldsymbol{x})$, such that $\nabla\phi(\boldsymbol{x}) \equiv \pi\Pi(\boldsymbol{x})$, the generation of topological defects in the field $\theta(\boldsymbol{x})$ can be described by a term

$$S_{ps} = -\sum_{n=1}^{\infty} \frac{\lambda_{ps}^n u}{\xi_0^2} \int d\boldsymbol{x} \cos[2n\phi(\boldsymbol{x})], \tag{12}$$

where $\lambda_{ps} = \exp\{-S_{\text{core}}\}$ is the "fugacity" of a phase slip and S_{core} is the action associated with the creation of a single phase slip.[30,33] The term $\cos[2n\phi(\boldsymbol{x})]$ represents the creation of a kink of value $2\pi n$ in the θ field at the space-time point $\boldsymbol{x}$. Assuming that $\lambda_{ps} \ll 1$, we can neglect in the following contributions with $n > 1$ in S_{ps}.

Adding Eqs. (6), (11), and (12) we finally arrive at the expression of the effective phase-only action at low temperatures

$$S = S_0 + S_A + S_{ps}, \tag{13}$$

describing on an equal footing the effects of fluctuation, dissipation, and topological excitations.

III. RESULTS

A. Renormalization-group analysis

To study the properties of the model of Eq. (13) at $T=0$, we perform a RG analysis which is perturbative in the couplings G_A and λ_{ps}. At lowest possible order, the RG equations are found by performing one-loop and two-loop corrections in S_A and S_{ps}, respectively.

We adopt a renormalization procedure that rescales space and time homogeneously, so as to preserve the Lorentz invariance of S_0. The renormalization of S_A involves a projection onto the most relevant sector of the coupling kernel $\tilde{P}_c(\boldsymbol{x})$, which is very anisotropic in space and time, obeying a functional RG flow in the general case. We can simplify the analysis by studying different scales of interest in the renormalization procedure. Depending on the final scale $\Lambda(l) \sim L^{-1}$ [where $\Lambda(l) = \Lambda_0 e^l$ is the renormalized momentum cutoff and where $\Lambda_0^{-1} = \xi_0$], we focus on the local part of the cooperon for $\Lambda(l) < \xi_N^{-1}$ or on the nonlocal, diffusive properties for $\Lambda(l) > \xi_N^{-1}$.

We can motivate the RG analysis in the nonlocal regime by noting that the kernel $\tilde{P}_c(\boldsymbol{x})$ induce Josephson coupling of phases over spatial distances $W(\tau) \sim \sqrt{D\tau}$. Indeed, an *effective* purely local kernel $\tilde{P}_c^{\text{eff}}(\boldsymbol{x})$ can be obtained by integrating the spatial coordinate in Eq. (10), yielding at long times

$$\tilde{P}_c^{\text{eff}}(\boldsymbol{x}) \sim \tau^{-2} W(\tau) \delta(x) \quad \text{for} \quad \tau \gg D/u^2. \tag{14}$$

This approximate form is simpler to analyze and yields a scaling dimension $\frac{3}{2}$. Note that this long-range temporal $\tau^{-3/2}$ differs from the standard local ohmic coupling τ^{-2} coupling[7] and further quenches fluctuations of the phase. A more detailed functional RG procedure involves an expansion of $\tilde{P}_c(\boldsymbol{x})$ in terms of Legendre polynomials and allows to extract the scaling dimension of $\tilde{P}_c$ in the nonlocal limit in a systematic way (cf. Appendix). Using that $\langle \cos[\theta(\boldsymbol{x}) - \theta(0)] \rangle \sim r^{-1/2K}$ for $r \to \infty$ (cf. Ref. 2), we conclude that the scaling dimension of the perturbative term S_A is $\frac{3}{2} - \frac{1}{2K}$.

In the local regime and for $\tilde{\lambda}_{ps} = 0$, our the RG analysis reduces to that obtained in Ref. 17, where details of their

derivation can be found. In this case, the scaling analysis of S_A is simpler to obtain since in this limit $\tilde{P}_c(\boldsymbol{x})$ is RG invariant, yielding a scaling dimension of $1-\frac{1}{2K}$.

We obtain the flow equations

$$\frac{dK(l)}{dl} = \tilde{G}_A(l) - \tilde{\lambda}_{ps}^2(l)K^3(l), \tag{15}$$

$$\frac{du(l)}{dl} = \tilde{G}_A(l)\frac{u(l)}{K(l)}\frac{B^{(x)}-B^{(\tau)}}{B^{(x)}+B^{(\tau)}}, \tag{16}$$

$$\frac{d\tilde{G}_A(l)}{dl} = \begin{cases} \left(1-\dfrac{1}{2K(l)}\right)\tilde{G}_A(l) & \text{(local)}, \\ \left(\dfrac{3}{2}-\dfrac{1}{2K(l)}\right)\tilde{G}_A(l) & \text{(nonlocal)}, \end{cases} \tag{17}$$

$$\frac{d\tilde{\lambda}_{ps}(l)}{dl} = [2-K(l)]\tilde{\lambda}_{ps}(l), \tag{18}$$

where we have defined the dimensionless couplings $\tilde{G}_A \equiv G_A\pi(B^{(x)}+B^{(\tau)})$ and $\tilde{\lambda}_{ps}\equiv\lambda_{ps}\sqrt{A}$ for convenience. The dimensionless quantities A and $B^{(x,\tau)}$ are nonuniversal and arise from the renormalization of S_{ps} and S_A, respectively, at scales $\{x,u\tau\}<\Lambda^{-1}(l)$ and are defined as

$$A \equiv \frac{1}{4\pi}\int_{\Lambda^{-1}(l)}^{\infty} d\tilde{r}\tilde{r}^3 e^{-2KF_\Lambda(\tilde{r})}F_\Lambda(\tilde{r}),$$

$$B^{(x)} \equiv \frac{1}{2}\frac{1}{\Lambda^2(l)u^2}\int_0^{2\pi} d\phi\tilde{P}_c(\Lambda^{-1}(l),\phi)\cos^2\phi,$$

$$B^{(\tau)} \equiv \frac{1}{2}\frac{1}{\Lambda^2(l)u^2}\int_0^{2\pi} d\phi\tilde{P}_c(\Lambda^{-1}(l),\phi)\sin^2\phi,$$

where $\tilde{r}\equiv\Lambda(l)\sqrt{x^2+(u\tau)^2}$ and $F_\Lambda(\boldsymbol{x})=\frac{1}{2}\ln\tilde{r}$, and where $\tilde{P}_c$ has been expressed in cylindrical coordinates [cf. Eq. (A8)]. It is interesting to point out that while only one parameter A arises in the rescaling of S_{ps} due to space-time isotropy, the anisotropy of S_A generates different parameters $B^{(x)}$ and $B^{(\tau)}$.

Note that in the local regime $[\tilde{P}_c\sim\tilde{P}_c(\tau)]$, the product uK does not renormalize for $\lambda_{ps}=0$ and thus $B^{(x)}=0$. Nonlocality is thus captured by a $B^{(x)}>0$. Further, since the term S_A breaks the space-time isotropy within our Lorentz-invariant RG analysis (i.e., momentum shell integration homogeneous in space time), we expect a renormalization of the velocity $u(l)$, cf. Eq. (16). A numerical evaluation gives $B^{(x)}/B^{(\tau)}<1$, meaning that $u(l)$ flows toward smaller values.

As for Eq. (18), we note that it corresponds to the usual BKT flow equation (cf. Ref. 30 for a derivation in the context of superconducting wires). In the limit $\{\tilde{G}_A(l),\tilde{\lambda}_{ps}(l)\}\to 0$, the properties of the system are dominated by the value of $K(l)$. From Eqs. (17) and (18), we can define the critical values $K_A^*\equiv\frac{1}{2}(\equiv\frac{1}{3})$ for the local (nonlocal) regime and $K_{ps}^*\equiv 2$. For $\tilde{\lambda}_{ps}=0$ and $K>K_A^*$, the coupling $\tilde{G}_A(l)$ flows toward strong coupling and eventually the perturbative RG analysis is no longer valid. On the other hand, for $\tilde{G}_A=0$ and $K<K_{ps}^*$ the coupling $\tilde{\lambda}_{ps}(l)$ becomes relevant and eventually superconductivity is destroyed in the wire, due to the unbinding of pairs of topological excitations.[2,30] Note that it is not possible to determine the nature of the $T=0$ fixed point in this regime within our formalism. This issue is currently under intensive research.[33] Therefore, at $T=0$ and when neither $\tilde{G}_A$ nor $\tilde{\lambda}_{ps}$ vanish, the Luttinger liquid phase is never stable and the ground state is determined by a competition between S_A and S_{ps}.

B. Self-consistent Harmonic approximation

To further investigate the properties in the regime where $\tilde{G}_A$ is the dominant parameter that flows to strong coupling, we used a self-consistent variational approach, the so-called self-consistent harmonic approximation.[2,47] This method consists in finding the optimal propagator $g_{\rm tr}(\boldsymbol{q})$ of a harmonic (Gaussian) trial action

$$S_{\rm tr}[\theta] = \frac{1}{2\beta L}\sum_{\boldsymbol{q}}\frac{1}{g_{\rm tr}(\boldsymbol{q})}|\theta(\boldsymbol{q})|^2$$

that minimizes the variational free energy

$$F_{\rm var} = F_{\rm tr} + T\langle S - S_{\rm tr}\rangle_{\rm tr},$$

where

$$F_{\rm tr} = -T\ln\int \mathcal{D}\theta e^{-S_{\rm tr}[\theta]}.$$

The minimization of the free-energy $F_{\rm var}$ with respect to $g_{\rm tr}(\boldsymbol{q})$ yields a self-consistent equation for $g_{\rm tr}(\boldsymbol{q})$

$$g_{\rm tr}^{-1}(\boldsymbol{q}) = g_0^{-1}(\boldsymbol{q}) - \frac{2G_A}{\xi_0^2}\int d\boldsymbol{x}[\cos(\boldsymbol{qx})-1] \times\tilde{P}_c(\boldsymbol{x})\exp\left\{-\frac{1}{\beta L}\sum_{\boldsymbol{q}'}[1-\cos(\boldsymbol{q}'\boldsymbol{x})]g_{\rm tr}(\boldsymbol{q}')\right\}, \tag{19}$$

where $g_0^{-1}(\boldsymbol{q})\equiv\frac{K}{\pi u}\omega_m^2+\frac{uK}{\pi}k^2$ is the propagator in the Luttinger liquid phase. The solutions of Eq. (19) read

$$g_{\rm tr}^{-1}(\boldsymbol{q}) = \begin{cases} g_0^{-1}(\boldsymbol{q})+\eta|\omega_m| & \text{(local)}, \\ g_0^{-1}(\boldsymbol{q})+\eta\sqrt{Dk^2+|\omega_m|} & \text{(nonlocal)}. \end{cases} \tag{20}$$

The parameter η is found self-consistently for the general case but in the limit $G_A\to 0$ it reduces to

$$\eta = \begin{cases} \left[\dfrac{2\pi G_A\xi_N\exp\left(\dfrac{\gamma}{2K}\right)}{\xi_0^2}\right]^{2K/2K-1}\left[\dfrac{\pi\xi_0}{4K}\right]^{1/2K-1} & \text{(local)}, \\ \left[\dfrac{8\pi^2 G_A\sqrt{D}}{\xi_0^2}\right]^{3K/3K-1}\left[\dfrac{\pi\xi_0^3 u}{4K\sqrt{D^3}}\right]^{1/3K-1} & \text{(nonlocal)}. \end{cases}$$

Note that physical solutions of the Eq. (19) with $\eta\neq 0$ are

found only for $K>K_A^*$, confirming the results of the RG analysis. In the context of the variational approach, it becomes clear [cf. Eq. (20)] that the contribution of the cooperon propagator of Eq. (11) induces ohmic (nonohmic) dissipation in the local (nonlocal) regime. Evaluation of the phase-correlation function at $T=0$ with the optimal $g_{\rm tr}(\boldsymbol{q})$ of Eq. (20) yields in the long-wavelength limit

$$\frac{\langle e^{i\theta(x)-i\theta(0)}\rangle}{\langle e^{i\theta}\rangle^2} \simeq \begin{cases} 1+\dfrac{1}{\sqrt{\pi}\eta}\dfrac{1}{x+\sqrt{\dfrac{8uK}{\pi\eta}\tau}} & \text{(local)}, \\ 1+\dfrac{2\sqrt{D}}{\eta\pi^2}\dfrac{1}{x^2+4D\tau} & \text{(nonlocal)}, \end{cases} \tag{21}$$

where $\langle e^{i\theta}\rangle$ is the value of the superconducting order parameter

$$\langle e^{i\theta}\rangle = \begin{cases} \left[\dfrac{\pi\xi_0\eta}{4K}\right]^{1/4K} & \text{(local)}, \\ \left[\dfrac{\pi\xi_0^3 u\eta}{4K\sqrt{D^3}}\right]^{1/6K} & \text{(nonlocal)}. \end{cases} \tag{22}$$

This result indicates that the order parameter develops long-range order and should be compared with the case of isolated wires, where superconducting correlation functions follow a power-law behavior and $\langle e^{i\theta}\rangle=0$ as a consequence of the strong quantum fluctuations.[1]

C. dc transport properties

Now we address the experimentally relevant question of the possibility to observe some signatures of our predictions at $T=0$. To that end, we turn our attention to transport properties and calculate the dc resistivity. We use the theoretical framework of the memory matrix, which is perturbative in the processes that degrade the current-density operator[2,29,48]

$$J(x) = \frac{uK}{\pi}\frac{2e}{c}\nabla\,\theta(x).$$

Current decay originated by phase slips induce finite resistivity at $T<T_c$. At very low temperatures $T\ll T_c$ thermally activated phase slips are suppressed and resistivity is dominated by quantum phase slips processes. In the absence of dissipation ($G_A=0$), its expression is well known and reads[29,30]

$$\varrho(T) \approx \frac{4\pi^3\tilde{\lambda}_{ps}^2\Lambda_0}{\left(\dfrac{2e}{c}\right)^2}B^2\left(\frac{K}{2},1-K\right)\cos^2\left(\frac{\pi K}{2}\right)\left(\frac{2\pi T}{u\Lambda_0}\right)^{2K-3}, \tag{23}$$

where $B(x,y)$ is the beta function. This is a valid expression provided that a perturbation expansion in λ_{ps} and G_A is possible. At finite temperatures, the effect of these couplings can be incorporated by replacing the bare parameters by the renormalized ones obtained from the integration of the RG-flow equations [Eqs. (15)–(18)] up to a scale[29] $\Lambda^{-1}(l)=u(l)/2\pi T$.

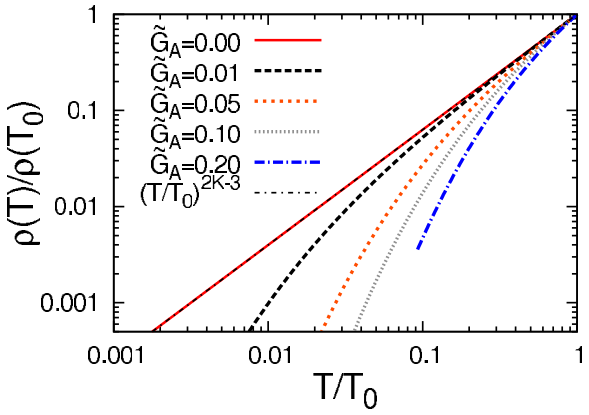

FIG. 2. (Color online) Normalized resistivity vs T/T_0. As the (dimensionless) Andreev conductance $\tilde{G}_A$ is increased, the wire resistivity ϱ (T) deviates from the law $\sim T^{2K-3}$ predicted for an isolated wire (Refs. 29, 30, and 33) as a consequence of the dissipation-induced increase in the stiffness K [cf. Eq. (15)].

Our results are shown in Fig. 2, where we calculate $\varrho(T)$ normalized to the "high-temperature" value $T_0=\xi_0/u$, fixed by the short-time cutoff of the theory. According with our estimations (see Sec. IV), we analyze only the local regime $\Lambda(l)\ll\xi_N^{-1}$. We start with the initial conditions $K(0)=2.1, \tilde{\lambda}_{ps}(0)=10^{-3}$, for $\tilde{G}_A(0)=0$ (solid line in Fig. 2). For comparison, we show the $(T/T_0)^{2K(0)-3}$ behavior predicted for the resistivity due to phase slips in the absence of dissipation effects (dot-dashed line).[29,30,33]

Interestingly, starting the RG flow with the initial values $\tilde{G}_A(0)=0.01$, 0.05, 0.1, and 0.2, the resistivity decreases *faster* than the $(T/T_0)^{2K(0)-3}$ law corresponding to the isolated wire. This illustrates the stabilizing effect of dissipation on superconductivity, which manifests itself through an increase in the stiffness K, as can be seen from Eq. (15) when parameter $\tilde{G}_A$ dominates over $\tilde{\lambda}_{ps}$. Note that since the integration of the renormalization-group flow (and consequently, the calculation of the resistivity) is perturbative, it cannot be carried beyond a point where either $\tilde{G}_A(l)$ or $\tilde{\lambda}_{ps}(l)$ become of order unity.

IV. DISCUSSION AND SUMMARY

The result of the RG flow Eqs. (15)–(18) together with the analysis in the strong-coupling regime, summarized in Eqs. (21) and (22), suggest that a weak coupling to the metallic film favors a superconducting ground state with long-range order of the order parameter at $T=0$, through a dissipation-induced quench of phase fluctuations.

Note that this is not trivial since a *strong* coupling to a disordered metallic film is detrimental to superconductivity and lowers T_c through the well-known proximity effect.[46] But in a low-dimensional situation at $T\ll T_c$, where phase fluctuations are the dominant mechanism of destruction of global phase coherence, the environment actually favors long-range order.

This picture is supported by experiments on disordered granular Pb films coated with a thin Ag metallic film,[49] where it was shown that while T_c *decreases* due to the proximity effect, phase stiffness actually *increases* at low enough temperatures. Also in the context of dissipative Josephson junctions arrays, it is well known that the existence of coupling to a normal metal stabilizes the superconducting phases.[7–11]

A similar idea has been recently suggested to produce an enhancement of T_c in bilayered materials[50] in which one layer has a high pairing scale but low superfluid density while in the other layer the situation is the inverse. When both materials are put into contact, the T_c of the coupled system is higher than those of the isolated layers. In the specific case of Luttinger liquids coupled to dissipative baths, our results are in agreement with recent theoretical works where the existence of superconductive long-range order at $T=0$ has been suggested.[17,18]

In this paper, we have presented a rigorous study of a realistic dissipative mechanism, provided by a coupling to a diffusive metal, in the context of superconducting wires. Of central importance in our analysis is the cooperon propagator kernel $\tilde{P}_c(\boldsymbol{x}-\boldsymbol{x}')$, which couples the field θ at the space-time coordinates $\boldsymbol{x}$ and $\boldsymbol{x}'$ in the dissipative term S_A [cf. Eq. (11)]. The physics of the kernel $\tilde{P}_c(\boldsymbol{x}-\boldsymbol{x}')$ strongly depends on the relation between $\xi_N(T)$, the coherence length in the diffusive film, and the length of the wire L. Consequently, two regimes of interest appear: the local regime $\xi_N(T) \ll L$, where the coupling of phases is purely local in space and nonlocal in time; and the nonlocal regime $\xi_N(T) > L$, where the phase coupling is nonlocal both in space and time. At this point, it is interesting to determine to which regime actual superconducting wires would correspond. At the experimentally relevant temperature $T \simeq 1K$ and using typical values of D in clean metallic films[51] $D \sim 10^2$ cm s^{-1}, we obtain the estimate $\xi_N(1K) \sim 0.1$ μm. On the other hand, the temperature constraints to observe nonlocal effects can be compactly written as $L_T(T) \ll L \ll \xi_N(T)$. These conditions require that $T \gg T_{NL} \equiv \hbar u^2/k_B D$ (where units have been restored) while the length has to be kept smaller than $\xi_N(T)$. Estimating the velocity of the Mooij-Schön as $u \sim 10^5$ m s^{-1} for the wires in Ref. 22, we obtain $T_{NL} \sim 10-100$ K, which exceeds the bulk-T_c values estimated in the range 6.9–7.1 K in In-Pb films.[23] The above estimates show that spatial nonlocal effects are elusive in actual wires (e.g., such as those studied in Ref. 27, where $L \sim 10-100$ μm) but may eventually be observed in superconducting wires with higher T_c, coupled to very clean substrates.

In order to make contact with recent transport experiments,[22–24,27,31] we have calculated the linear dc resistivity of a wire weakly coupled to a diffusive film for different values of the Andreev conductance G_A. The results of Fig. 2, calculated for the local regime $\xi_N(T) \ll L$, suggest that signatures of the predicted long-range order phase at $T=0$ could be observed experimentally. Indeed, since the dissipative term S_A renormalizes the superconducting stiffness K to higher values as the temperature decreases [cf. Eq. (15)], sizable deviations from the predictions for an isolated wire[29,30] $\varrho(T) \sim T^{2K(0)-3}$, where $K(0)$ is the bare stiffness, could be achieved at low enough temperatures.

In summary, we have studied a thin superconducting wire weakly coupled to a metallic film, focusing on the details of dissipation provided by the metallic cooperons at low temperatures. We have studied the phase diagram at $T=0$ within the framework of renormalization group and a variational harmonic approximation. We predict a quantum phase transition toward a superconductor with long-range order at $T=0$ as a function of the Andreev conductance G_A and the superconducting stiffness K of the wire. Finally, we show that some signatures of this ordered phase could be observed in experiments of transport, manifested as an increase in the superconducting stiffness and consequently the exponent of $\varrho(T) \sim T^{\nu}$ at low temperatures.

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APPENDIX

1. Cooperon propagator

In this section we derive the expression for the cooperon propagator assuming weak disorder in the metallic film. We refer the reader to Ref. 43 for further details.

When evaluating averages over the disorder potential in Eq. (8)

$$P_c(\boldsymbol{r}-\boldsymbol{r}',\tau-\tau') \equiv \langle g_{2D}(\boldsymbol{r},\boldsymbol{r}';\tau-\tau') g_{2D}(\boldsymbol{r},\boldsymbol{r}';\tau-\tau')\rangle_d$$

a diagrammatic series (ladder diagrams) is constructed upon the repeated action of the Dyson's equation (in operator notation)

$$\hat{\boldsymbol{g}}_{2D} = \hat{\boldsymbol{g}}^0_{2D} + \hat{\boldsymbol{g}}^0_{2D}\hat{\boldsymbol{V}}\hat{\boldsymbol{g}}_{2D},$$

where $\hat{\boldsymbol{g}}^0_{2D}$ is the unperturbed electron Green's function in the otherwise perfect metal and $\delta(\boldsymbol{r}-\boldsymbol{r}')V(\boldsymbol{r}) = \langle \boldsymbol{r}|\hat{V}|\boldsymbol{r}'\rangle$ is the (static) disorder potential which verifies

$$\langle V(\boldsymbol{r})\rangle_d = 0, \tag{A1}$$

$$\langle V(\boldsymbol{r})V(\boldsymbol{r}')\rangle_d = n_i \mathcal{V}^2 \delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{A2}$$

where n_i is the concentration of impurities and $\mathcal{V}$ is the uniform component of $V(\boldsymbol{r})$ in Fourier space. The diagrammatic series in Fourier space representation is given by (cf. Ref. 43)

$$P_c(\boldsymbol{Q};n,m) = \frac{P^0_c(\boldsymbol{Q};n,m)}{1-\dfrac{n_i\mathcal{V}^2}{\Omega}P^0_c(\boldsymbol{Q};n,m)},$$

where

$$P^0_c(\boldsymbol{Q};n,m) \equiv \sum_{\boldsymbol{p}} g_{2D}(\boldsymbol{p},i\nu_n) g_{2D}(\boldsymbol{Q}-\boldsymbol{p},i\nu_m)$$

and where Ω is the volume of the sample. $\boldsymbol{Q}=\boldsymbol{k}+\boldsymbol{k}'$ represents the center-of-mass momentum of the two electron sys-

tem, and $\boldsymbol{k}$ and $\boldsymbol{k}'$ are the initial (i.e., before colliding with impurities) momenta of the individual electrons. Defining

$$i\omega_l \equiv i\nu_m - i\nu_n,$$

$$\zeta(\boldsymbol{Q};i\nu_n,i\omega_l) \equiv \frac{n_i\mathcal{V}^2}{\Omega}P_c^0(\boldsymbol{Q};n,l+n),$$

the real-space representation of P_c reads

$$P_c(\boldsymbol{r},\tau) = \left(\frac{1}{\beta\Omega}\right)^2 \sum_{\boldsymbol{Q}=\boldsymbol{k}+\boldsymbol{k}'} \sum_{n,l} e^{-i(2\nu_n+\omega_l)\tau} e^{i\boldsymbol{Q}\cdot\boldsymbol{r}} \times \frac{\frac{\Omega}{n_i\mathcal{V}^2}\zeta(\boldsymbol{Q};i\nu_n,i\omega_l)}{1-\zeta(\boldsymbol{Q};i\nu_n,i\omega_l)}. \quad \text{(A3)}$$

Using the definition of the mean-free path $l_e \equiv v_F\tau_e$ and the definition of the elastic lifetime (in Born's approximation)[43]

$$\frac{1}{\tau_e} \equiv 2\pi\rho_{2D}n_i\mathcal{V}^2, \quad \text{(A4)}$$

we obtain in the diffusive limit ($Ql_e, \omega_l\tau_e \ll 1$),

$$\zeta(Q;i\nu_n,i\omega_l) \approx 1 - \tau_e(|\omega_l| + DQ^2),$$

where we have defined the diffusion constant in d spatial dimensions $D \equiv \frac{v_F^2\tau_e}{d} = \frac{l_e^2}{\tau_e d}$. Replacing these results into Eq. (A3) and noting that the contribution to $\zeta(\boldsymbol{Q};i\nu_n,i\omega_l)$ is vanishingly small for $\nu_n(\nu_n+\omega_l) > 0$, we obtain

$$P_c(\boldsymbol{r},\tau) \approx 2\pi\rho_{2D}\frac{1}{\beta}\sum_{\nu_n} e^{-i2\nu_n\tau} \times \left.\frac{1}{\beta\Omega}\sum_{\omega_l,\boldsymbol{Q}} \frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}e^{-i\omega_l\tau}}{|\omega_l| + DQ^2}\right|_{\nu_n(\nu_n+\omega_l)<0}. \quad \text{(A5)}$$

In addition, and since the condition $\nu_n(\nu_n+\omega_l) < 0$ must be fulfilled, we note that the limit $|\omega_l|\tau_e \to 0$ constrains the summation over Matsubara frequencies ν_n to values near $\nu_n \approx 0$. Therefore, at low temperatures we find

$$P_c(\boldsymbol{r},\tau) = 2\rho_{2D}\frac{1}{\beta\Omega}\sum_{\omega_l,\boldsymbol{Q}} \frac{\sin(|\omega_l|\tau)}{\tau}\frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}}{|\omega_l| + DQ^2}.$$

We can generalize this expression to take into account processes that break phase coherence (magnetic fields, magnetic impurities, etc.)

$$P_c(\boldsymbol{r},\tau) = 2\rho_{2D}\frac{1}{\beta\Omega}\sum_{\omega_l,\boldsymbol{Q}} \frac{\sin(|\omega_l|\tau)}{\tau}\frac{e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}}{|\omega_l| + DQ^2 + \tau_\varphi^{-1}}, \quad \text{(A6)}$$

where τ_φ is a phenomenological phase-breaking time. At $T=0$ we can replace $\frac{1}{\beta}\Sigma_{\omega_l} \to \frac{1}{2\pi}\int_0^\infty d\omega$. Then, the sum over $\boldsymbol{Q}$ in Eq. (A6) yields for a 2D system

$$P_c(\boldsymbol{r},\tau) = \mathrm{Re}\left\{\frac{\rho_{2D}}{i\tau\pi^2 D}\int_0^\infty d\omega e^{\omega(i\tau-\tau_c)}K_0\left(\sqrt{\frac{\omega+\tau_\varphi^{-1}}{D}}r\right)\right\},$$

where $K_0(x)$ is the zeroth-order modified Bessel function. $r = \sqrt{x^2+y^2}$ is the distance in the x-y plane and τ_c is an ultraviolet cutoff in time to make the integral in ω convergent, and which we set $\tau_c = \tau_e$. Finally we arrive at the expression of the cooperon in Eq. (9)

$$P_c(\boldsymbol{r},\tau) = \frac{\rho_{2D}}{2\pi^2 D}\tilde{P}_c(\boldsymbol{r},\tau), \quad \text{(A7)}$$

where we defined

$$\tilde{P}_c(\boldsymbol{r},\tau) = \mathrm{Re}\left\{\frac{ie^{-r/\xi_N}e^{ir^2/4D\tilde{\tau}}\Gamma\left[0,\frac{ir^2}{4D\tilde{\tau}}\right]}{\tilde{\tau}^2}\right\},$$

$$\tilde{\tau} = \tau + i\tau_e$$

and where $\Gamma[\alpha,z]$ is the incomplete gamma function.

2. Expansion in terms of Legendre polynomials

In order to investigate the scaling dimensions of the bare kernel $\tilde{P}_c$ in the nonlocal regime, we can use the approximate expression of Eq. (10)

$$\tilde{P}_c(\boldsymbol{x}) \approx \frac{(4D)^2}{x^4 + (4D\tau)^2}$$

and express it in terms of cylindrical coordinates $\{x = r\cos\phi, u\tau = r\sin\phi\}$ as

$$\tilde{P}_c(\boldsymbol{x}) = \tilde{P}_c(r,\phi) \approx \frac{(4D)^2}{r_D^4}\frac{1}{\tilde{r}^2}\tilde{f}(\tilde{r},\phi), \quad \text{(A8)}$$

where the definitions

$$\tilde{f}(\tilde{r},\phi) \equiv \frac{1}{\tilde{r}^2\cos^4\phi + \sin^2\phi}, \quad \text{(A9)}$$

$$\tilde{r} \equiv \frac{r}{r_D}, \quad \text{(A10)}$$

$$r_D \equiv \frac{4D}{u} = \frac{4v_F}{u}l_e \quad \text{(A11)}$$

have been used and where v_F is the Fermi velocity in the metallic film. We now expand $\tilde{f}(\tilde{r},\phi)$ in Legendre polynomials in order to separate radial and angular variables in a systematic way

$$\tilde{f}(\tilde{r},\phi) = \sum_{\ell=0}^{\infty} A_\ell(\tilde{r})\mathcal{P}_\ell(\cos\phi),$$

where the coefficients are defined as

$$A_\ell(\tilde{r}) \equiv \frac{2\ell+1}{2}\int_{-1}^{1} d(\cos\phi)\tilde{f}(\tilde{r},\phi)\mathcal{P}_\ell(\cos\phi). \quad \text{(A12)}$$

Then,

$$\tilde{P}_c(r,\phi) = \frac{(4D)^2}{r_D^4}\frac{1}{\tilde{r}^2}\sum_{\ell=0}^{\infty} A_\ell(\tilde{r})\mathcal{P}_\ell(\cos\phi). \tag{A13}$$

Changing variables to $\nu \equiv \cos\phi$, we can write Eq. (A12) as

$$A_\ell(\tilde{r}) = \frac{2\ell+1}{2}\int_{-1}^{1} d\nu \mathcal{P}_\ell(\nu)\tilde{f}(\tilde{r},\nu)$$

$$= \frac{2\ell+1}{2}\int_{-1}^{1} d\nu \frac{\mathcal{P}_\ell(\nu)}{\tilde{r}^2\nu^4 - \nu^2 + 1}. \tag{A14}$$

An asymptotic approximation of $A_\ell(\tilde{r})$ in the regime $\tilde{r}\to\infty$ gives

$$\lim_{\tilde{r}\to\infty} A_\ell(\tilde{r}) \to \frac{2\ell+1}{2}\mathcal{P}_\ell(0)\int_{-1}^{1} d\nu \frac{1}{\tilde{r}^2\nu^4+1},$$

$$\to \frac{2\ell+1}{2}\mathcal{P}_\ell(0)\frac{\pi}{\sqrt{2\tilde{r}}} \tag{A15}$$

where from the Rodrigues formula[52] we obtain $\mathcal{P}_\ell(0) = \frac{(-1)^{\ell/2}}{2^\ell}\frac{\ell!}{[(\ell/2)!]^2}$ for ℓ even [note from Eq. (A14) that $A_\ell(\tilde{r})$ vanish for odd values of ℓ]. Therefore, from Eqs. (A13) and (A15) we obtain in the asymptotic limit

$$\tilde{P}_c(r,\phi) \underset{r\to\infty}{\to} \frac{\pi}{\sqrt{2}}\frac{(4D)^2}{r_D^4}\frac{1}{\tilde{r}^{5/2}}\sum_{\ell=0}^{\infty}\frac{2\ell+1}{2}\mathcal{P}_\ell(0)\mathcal{P}_\ell(\cos\phi),$$

$$\underset{r\to\infty}{\to} \frac{\pi}{\sqrt{2}}\frac{(4D)^2}{r_D^4}\frac{1}{\tilde{r}^{5/2}}\delta(\cos\phi),$$

where in the last line the expansion of the Dirac-delta function in terms of the Legendre polynomials has been used. Coming back to coordinates x and τ, we obtain

$$\tilde{P}_c(\boldsymbol{x}) \underset{r\to\infty}{\to} \frac{\pi\sqrt{4D}}{\sqrt{2}}\frac{\delta(x)}{\tau^{3/2}},$$

which provides a rigorous derivation of Eq. (14).

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